Convolutional Neural Networks

Fundamentals of Data Science
15 December 2023
Prof. Fabio Galasso

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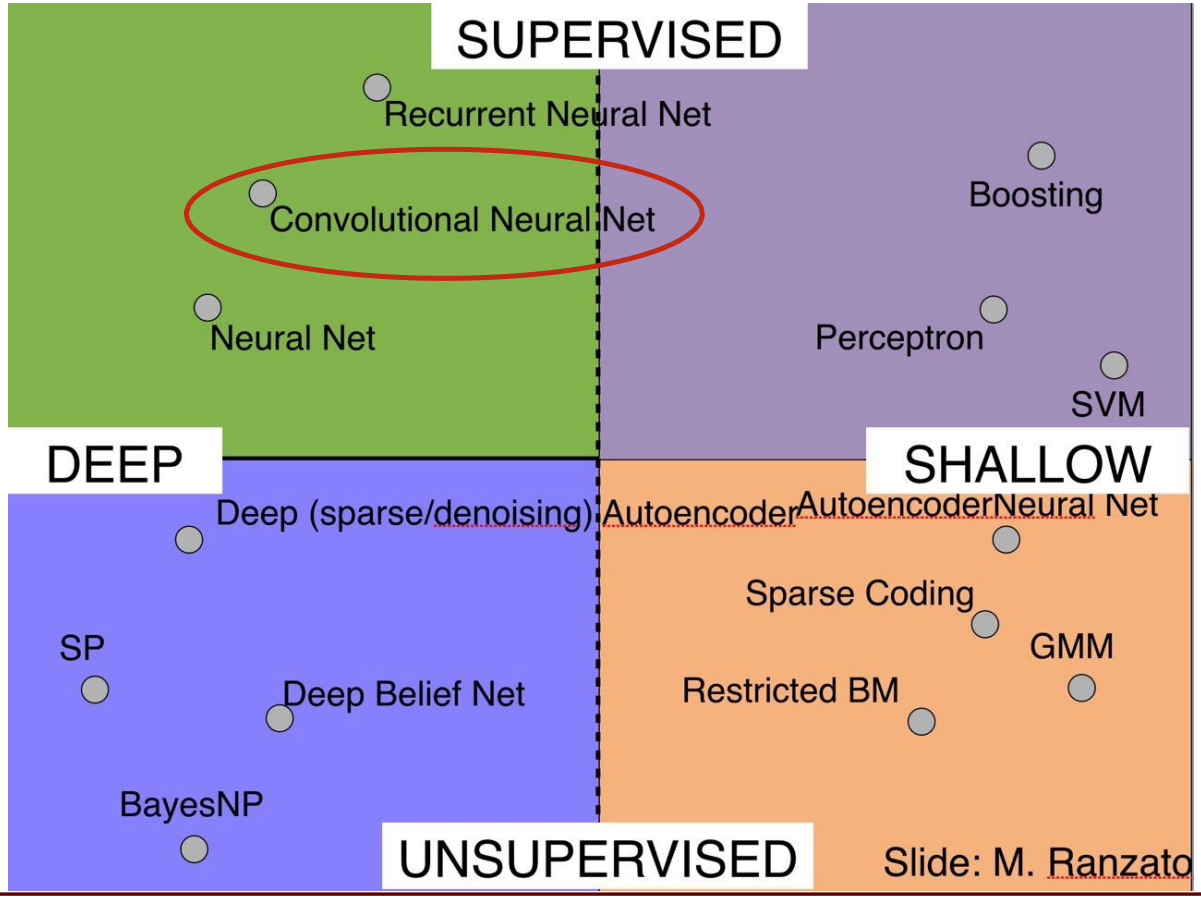
Overview of Neural Network Architectures



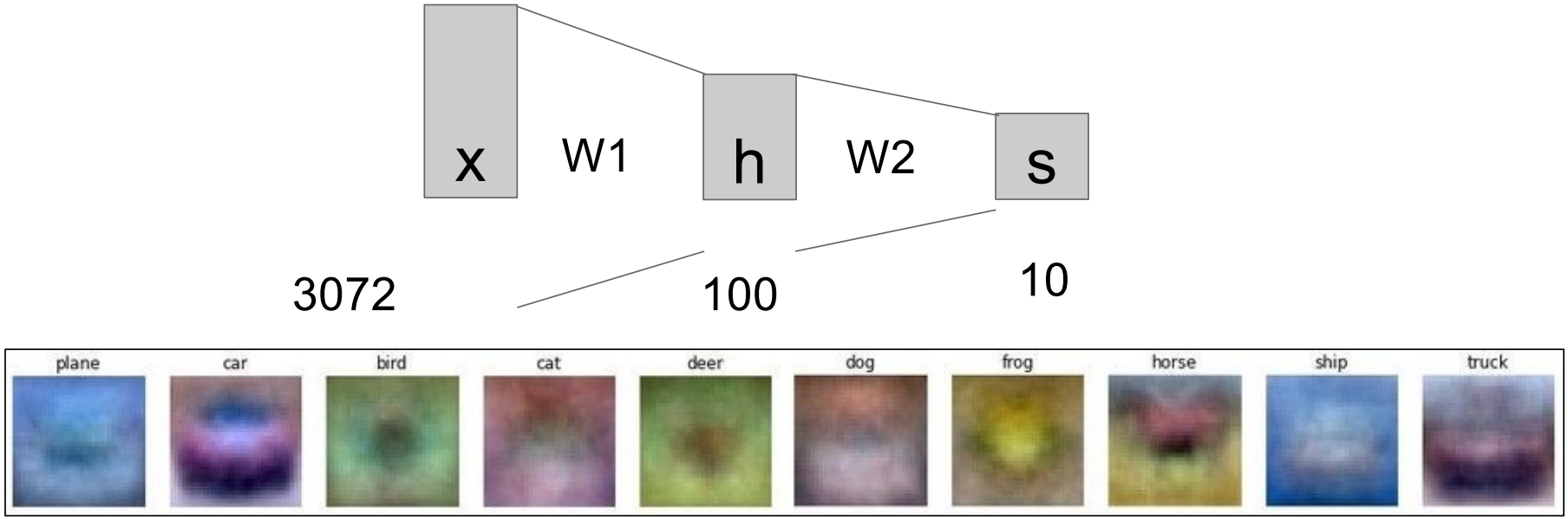
Neural Networks

Linear score function:

$$f = Wx$$

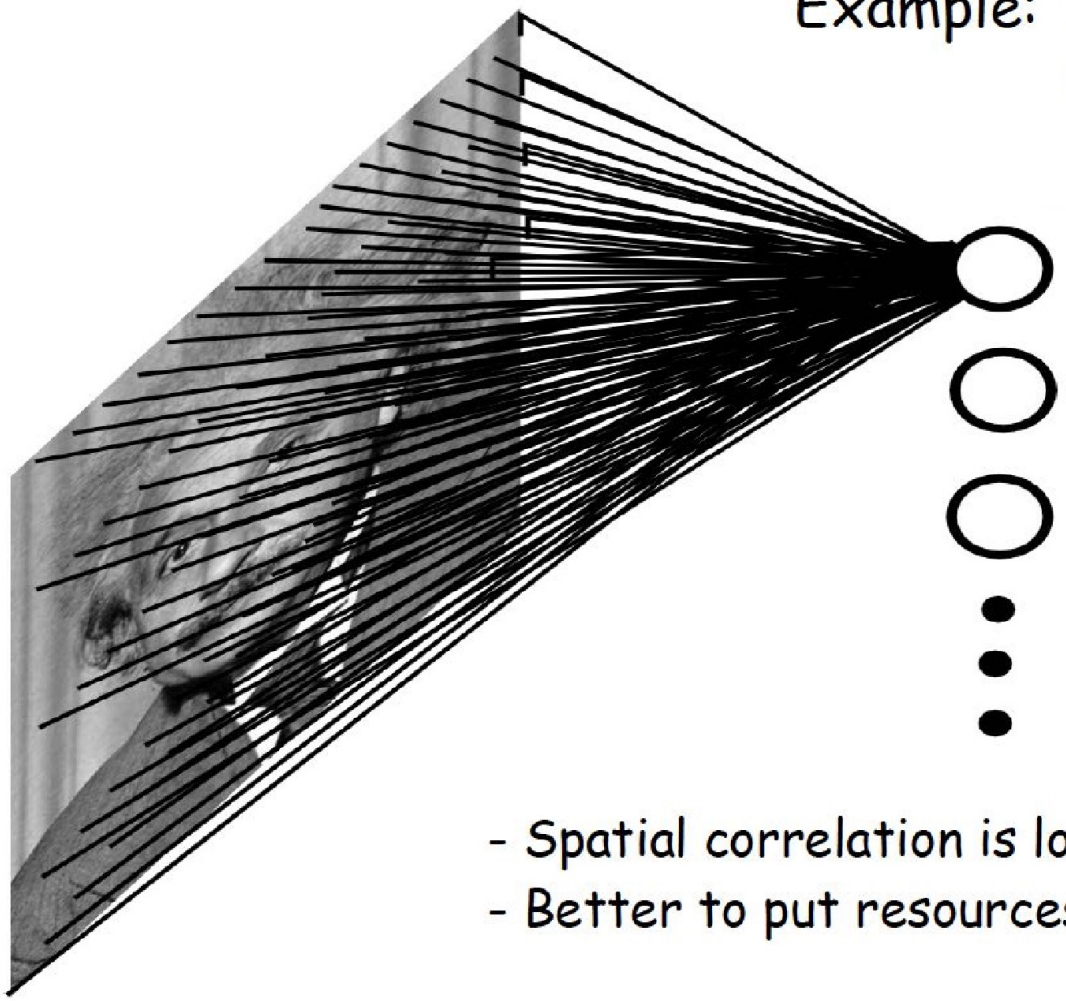
2-layer Neural Network

$$f = W_2 \max(0, W_1 x)$$



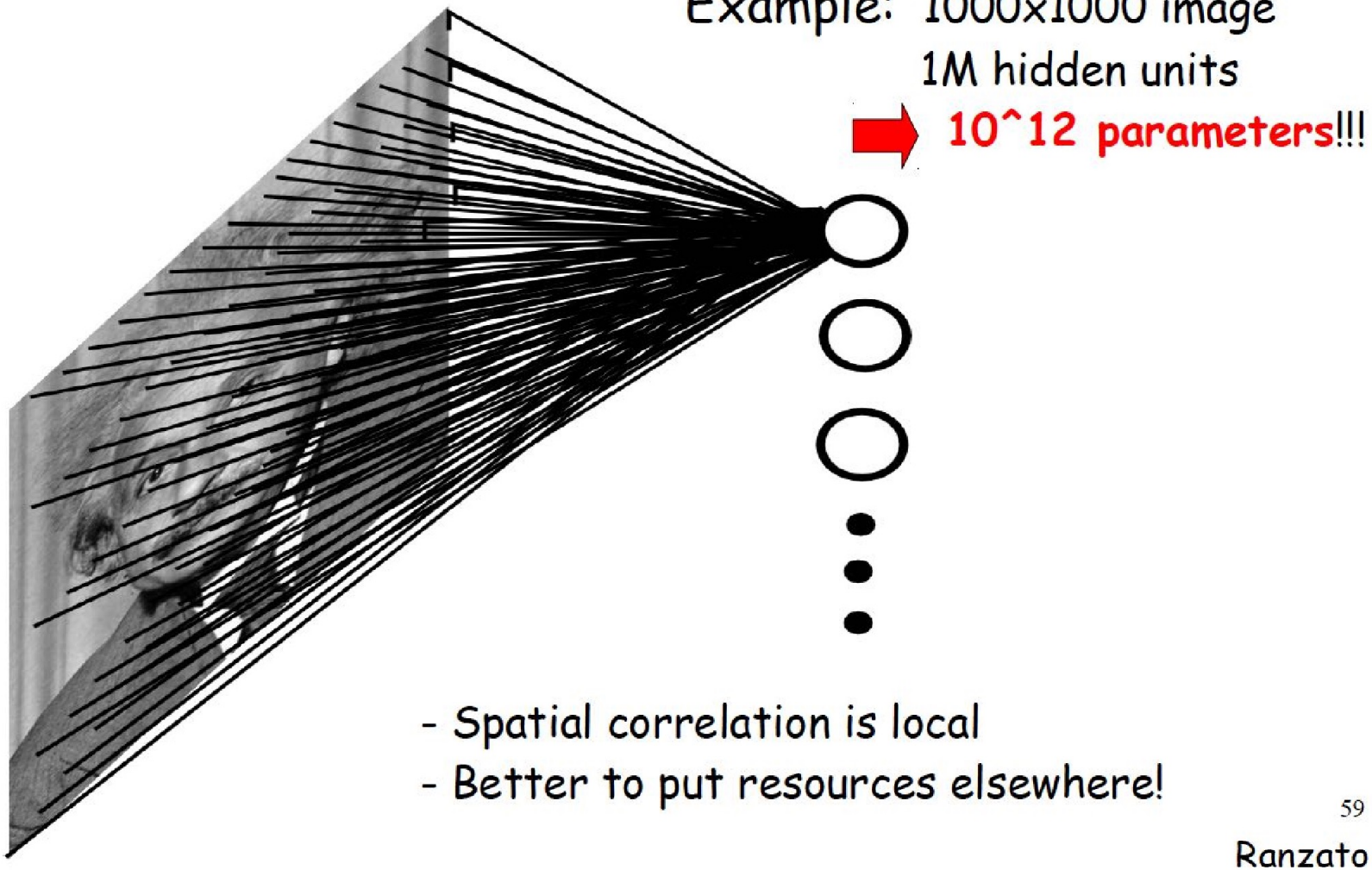
FULLY CONNECTED NEURAL NET

Example: 1000x1000 image
1M hidden units



- Spatial correlation is local
- Better to put resources elsewhere!

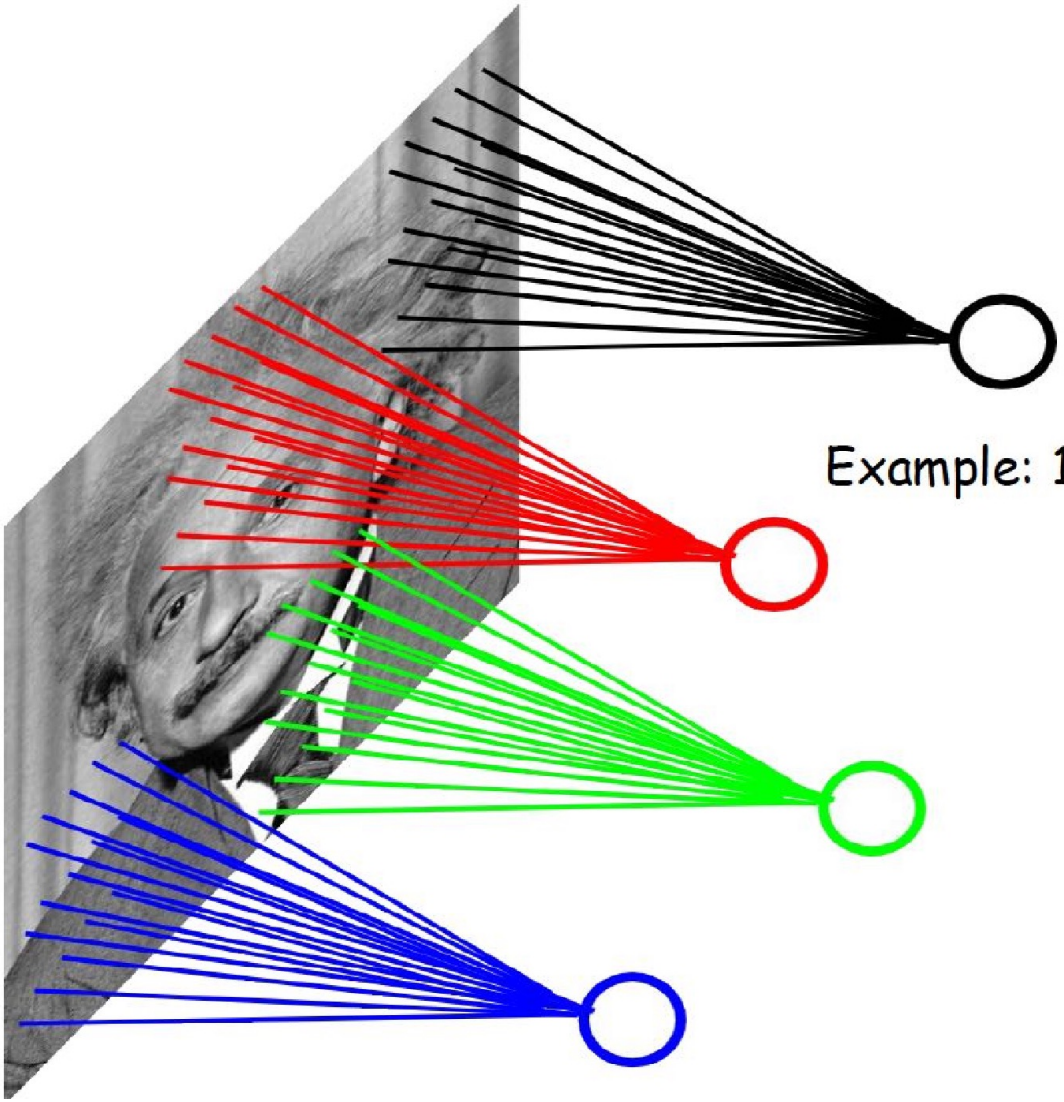
FULLY CONNECTED NEURAL NET



Example: 1000x1000 image
1M hidden units
10^12 parameters!!!

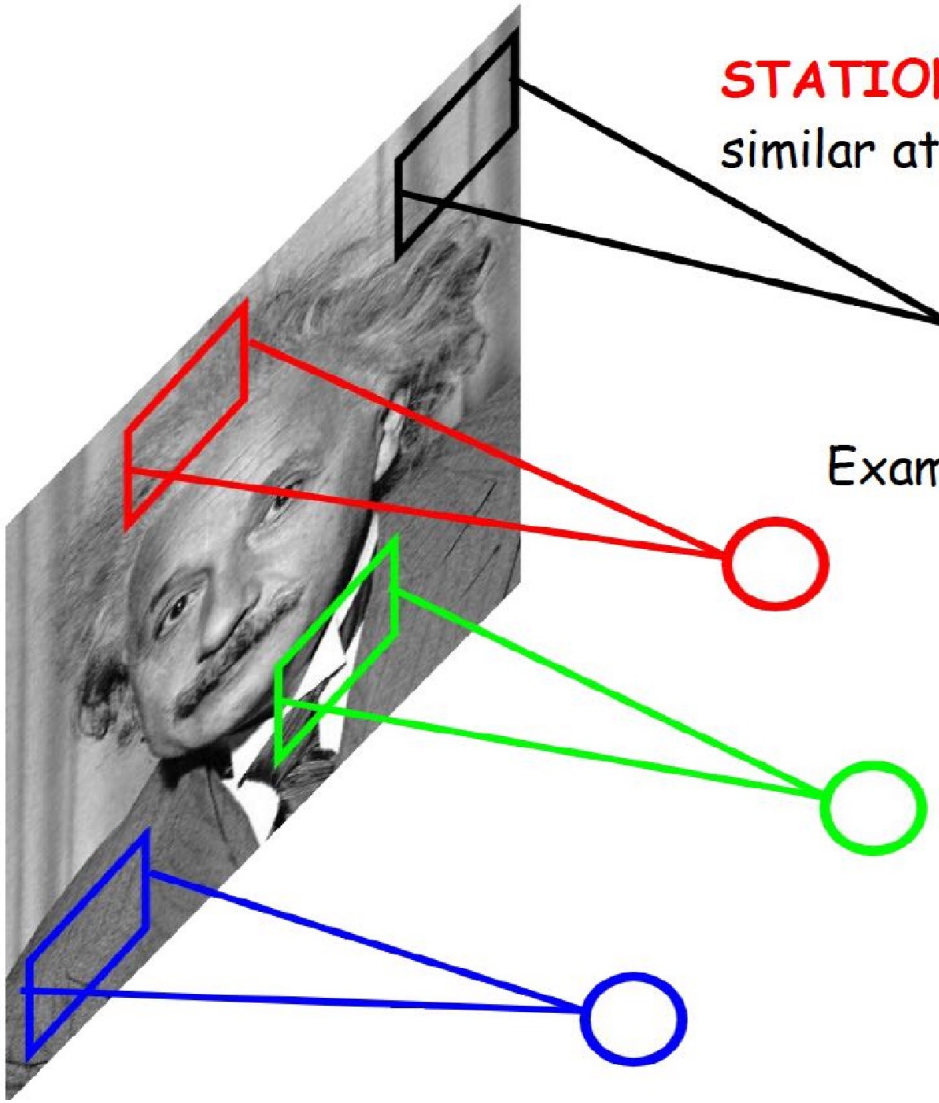
- Spatial correlation is local
- Better to put resources elsewhere!

LOCALLY CONNECTED NEURAL NET



Example: 1000x1000 image
1M hidden units
Filter size: 10x10
100M parameters

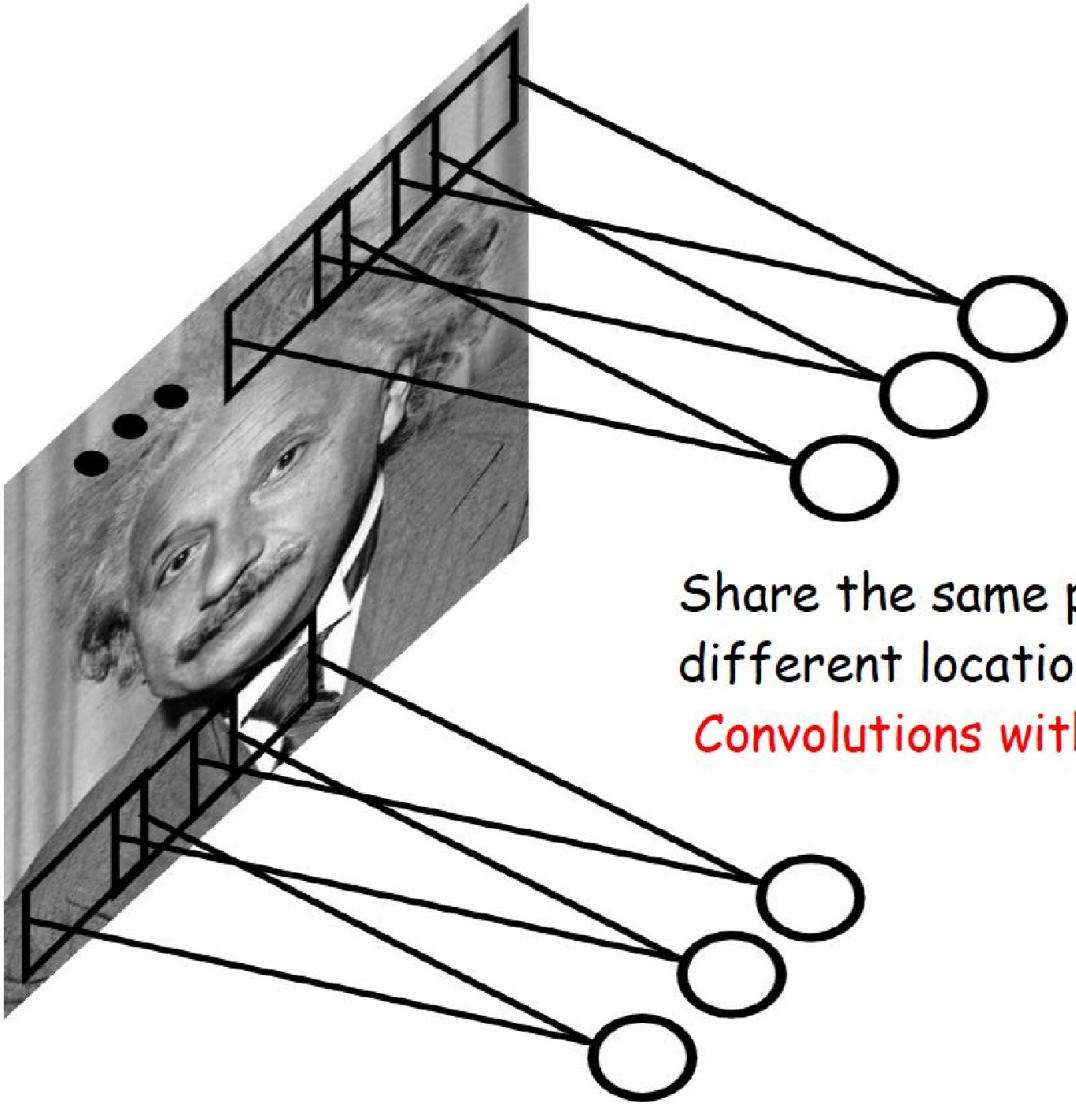
LOCALLY CONNECTED NEURAL NET



STATIONARITY? Statistics is similar at different locations

Example: 1000x1000 image
1M hidden units
Filter size: 10x10
100M parameters

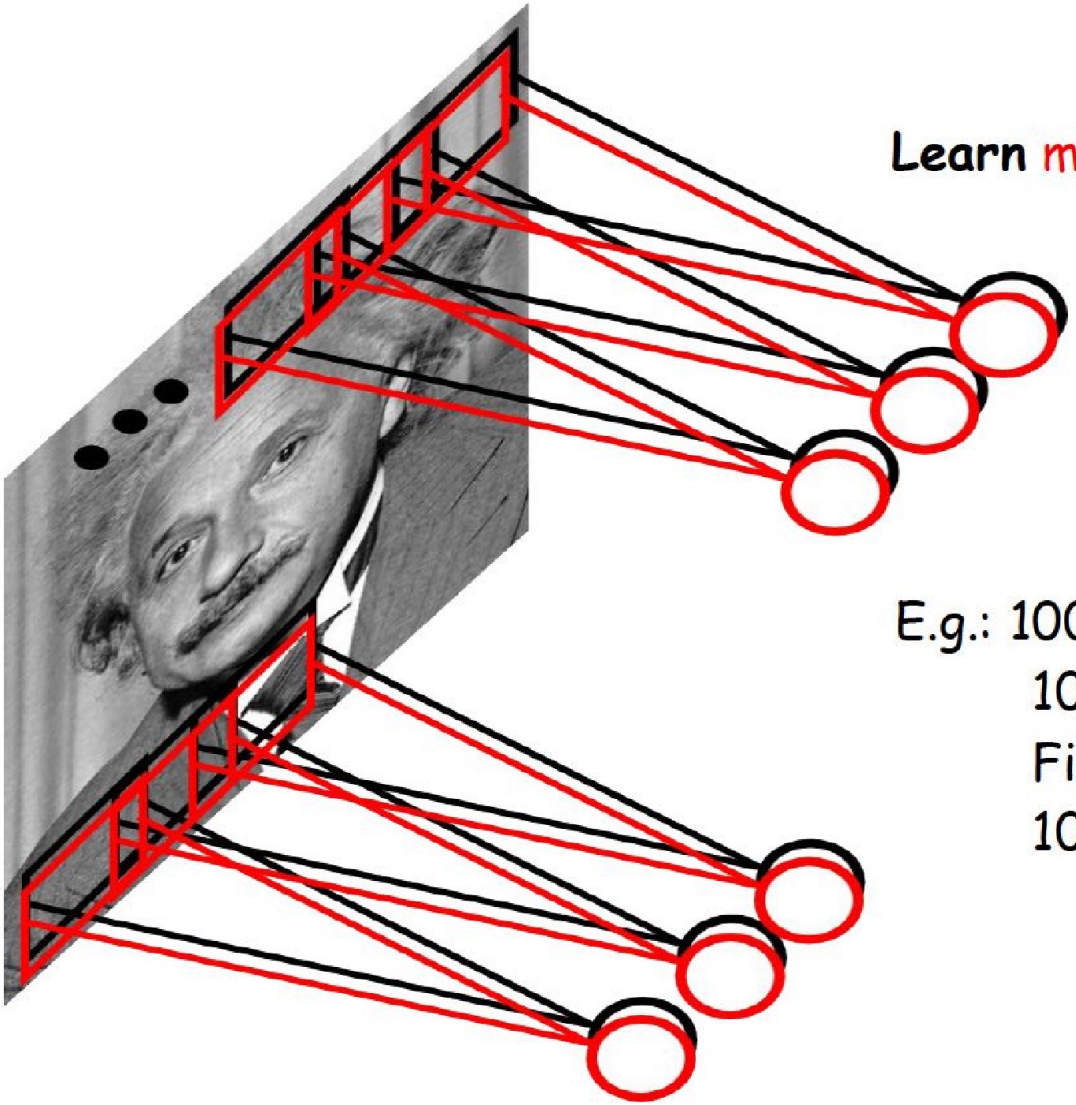
CONVOLUTIONAL NET



Share the same parameters across different locations:

Convolutions with learned kernels

CONVOLUTIONAL NET



Learn multiple filters.

E.g.: 1000x1000 image

100 Filters

Filter size: 10x10

10K parameters

NEURAL NETS FOR VISION

A standard neural net applied to images:

- scales quadratically with the size of the input
- does not leverage stationarity

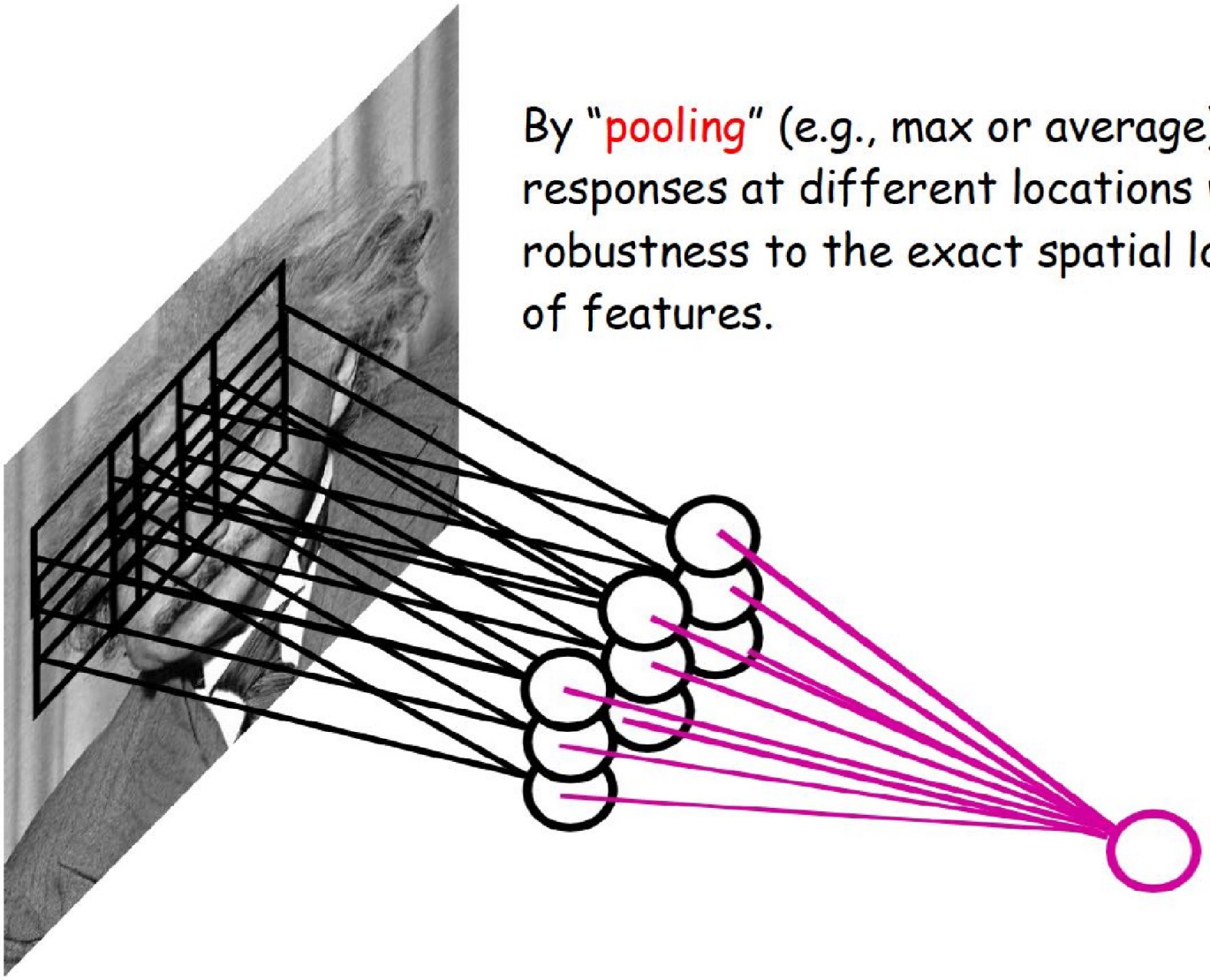
Solution:

- connect each hidden unit to a small patch of the input
- share the weight across hidden units

This is called: **convolutional network.**

LeCun et al. "Gradient-based learning applied to document recognition" IEEE 1998

CONVOLUTIONAL NET



By "pooling" (e.g., max or average) filter responses at different locations we gain robustness to the exact spatial location of features.

Convolutional Neural Networks

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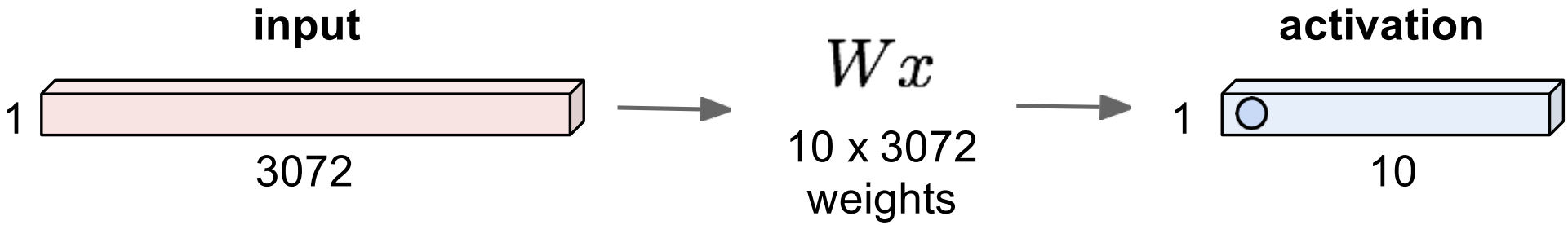
Fully Connected Layer

32x32x3 image -> stretch to 3072 x 1



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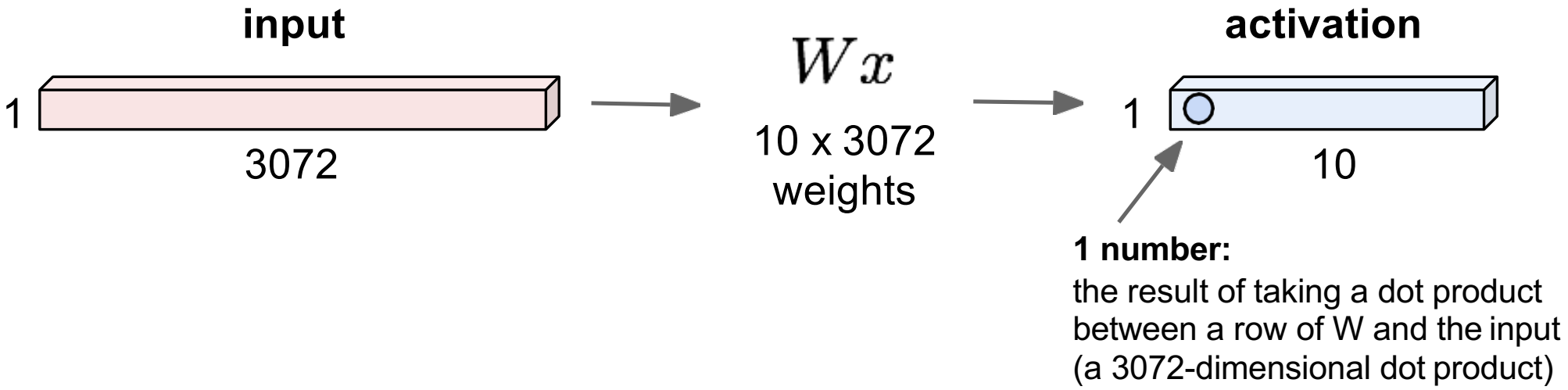
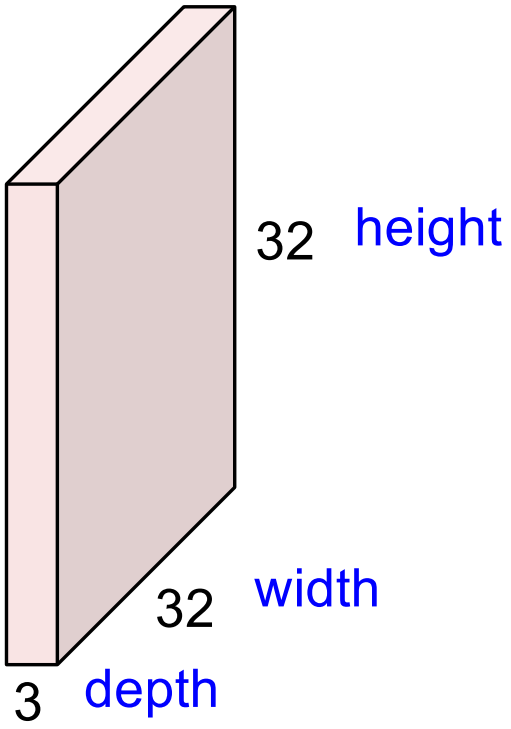


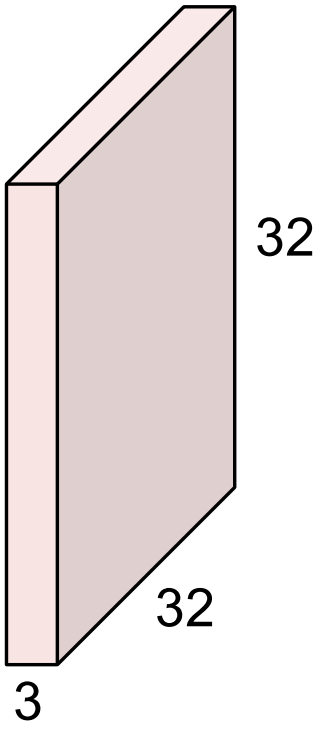
Convolution Layer

32x32x3 image -> preserve spatial structure

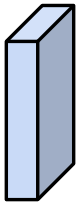


Convolution Layer

32x32x3 image



5x5x3 filter

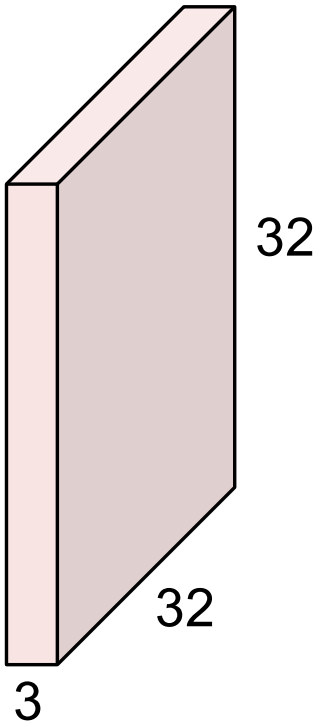


Convolve the filter with the image i.e. “slide over the image spatially, computing dot products”

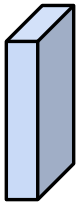
Convolution Layer

Filters always extend the full depth of the input volume

32x32x3 image



5x5x3 filter



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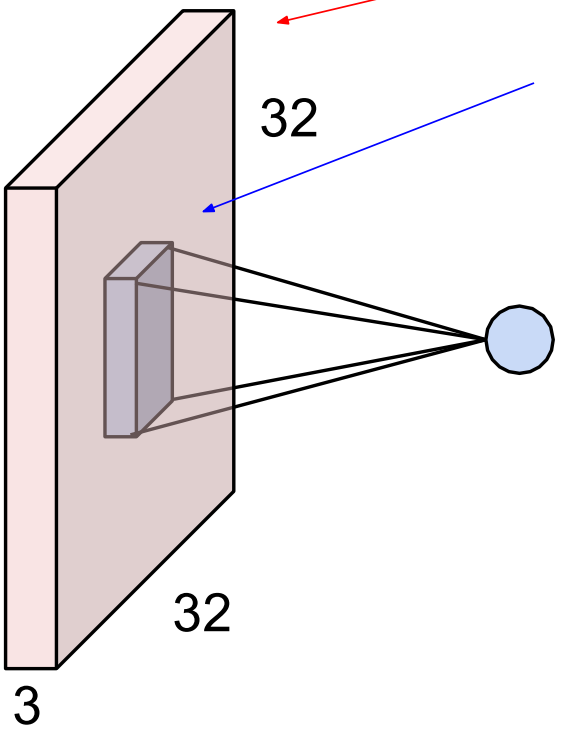
Convolution Layer



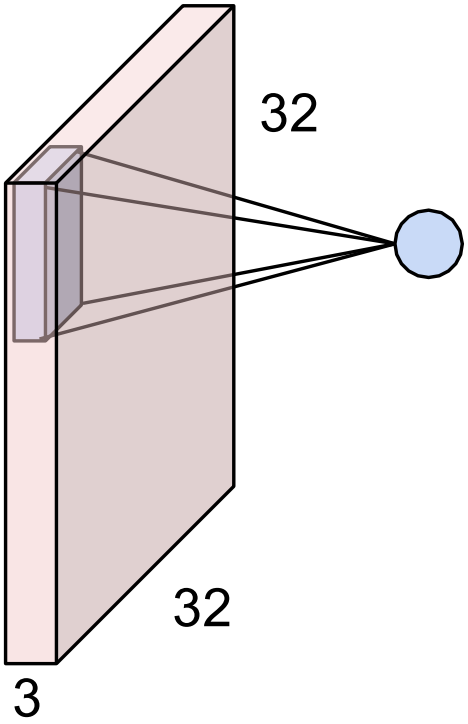
32x32x3 image

5x5x3 filter w

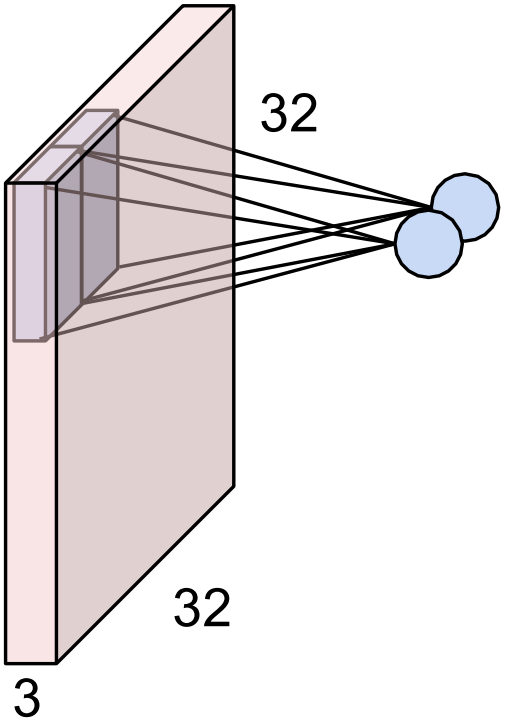
1 number:
the result of taking a dot product between the filter and a small 5x5x3 chunk of the image
(i.e. 5*5*3 = 75-dimensional dot product + bias)

$$w^T x + b$$

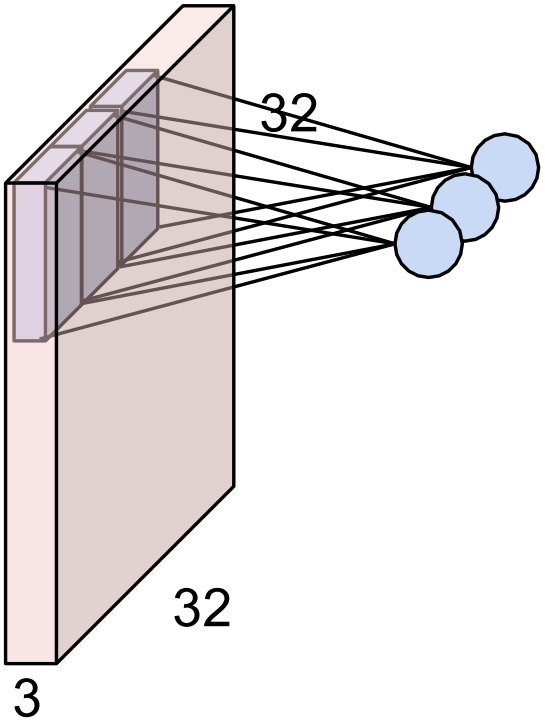
Convolution Layer



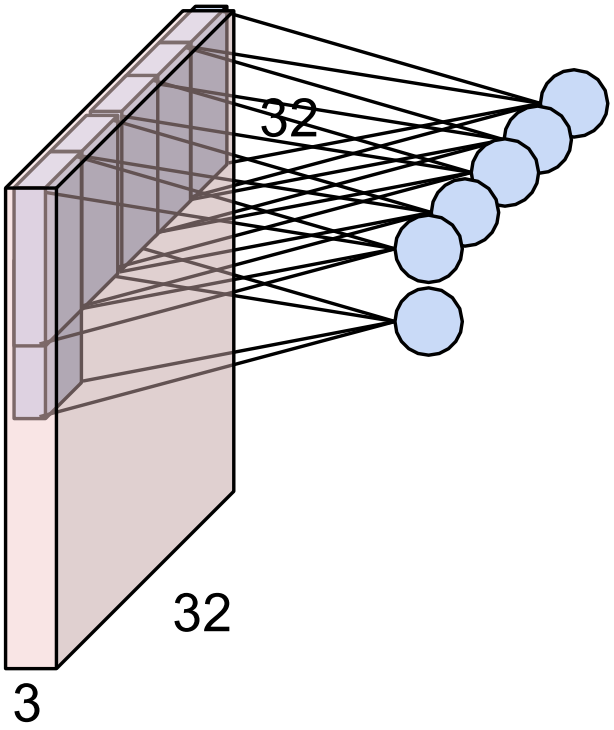
Convolution Layer



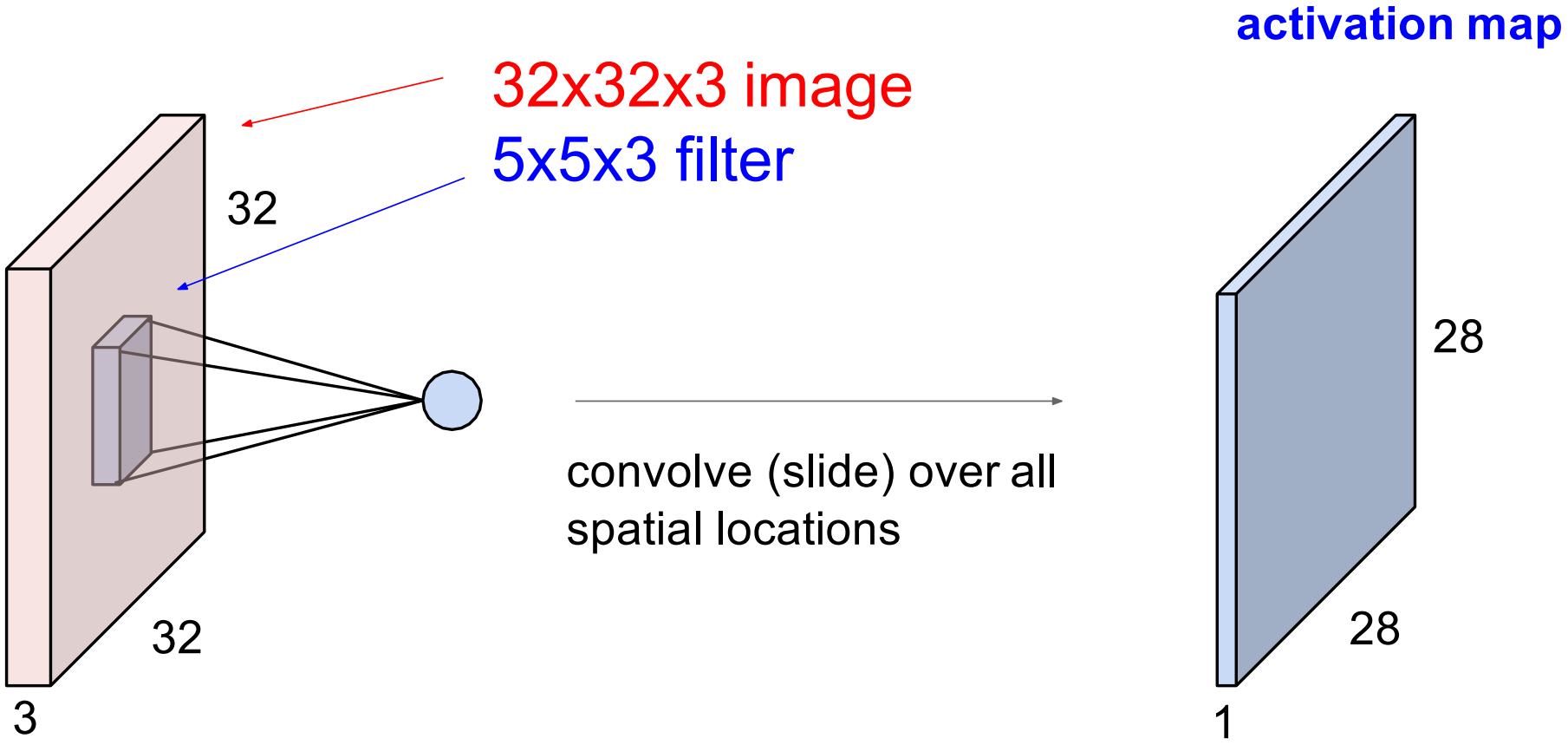
Convolution Layer



Convolution Layer

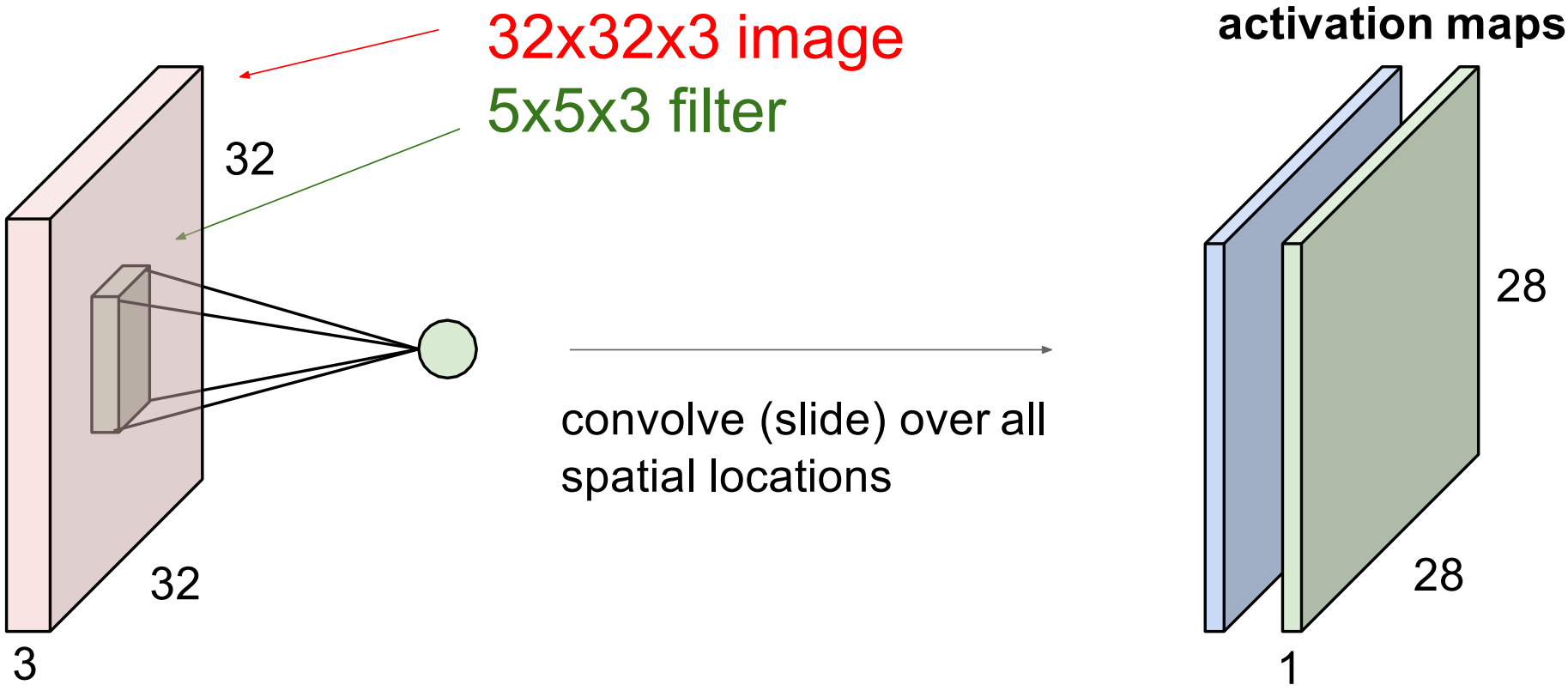


Convolution Layer

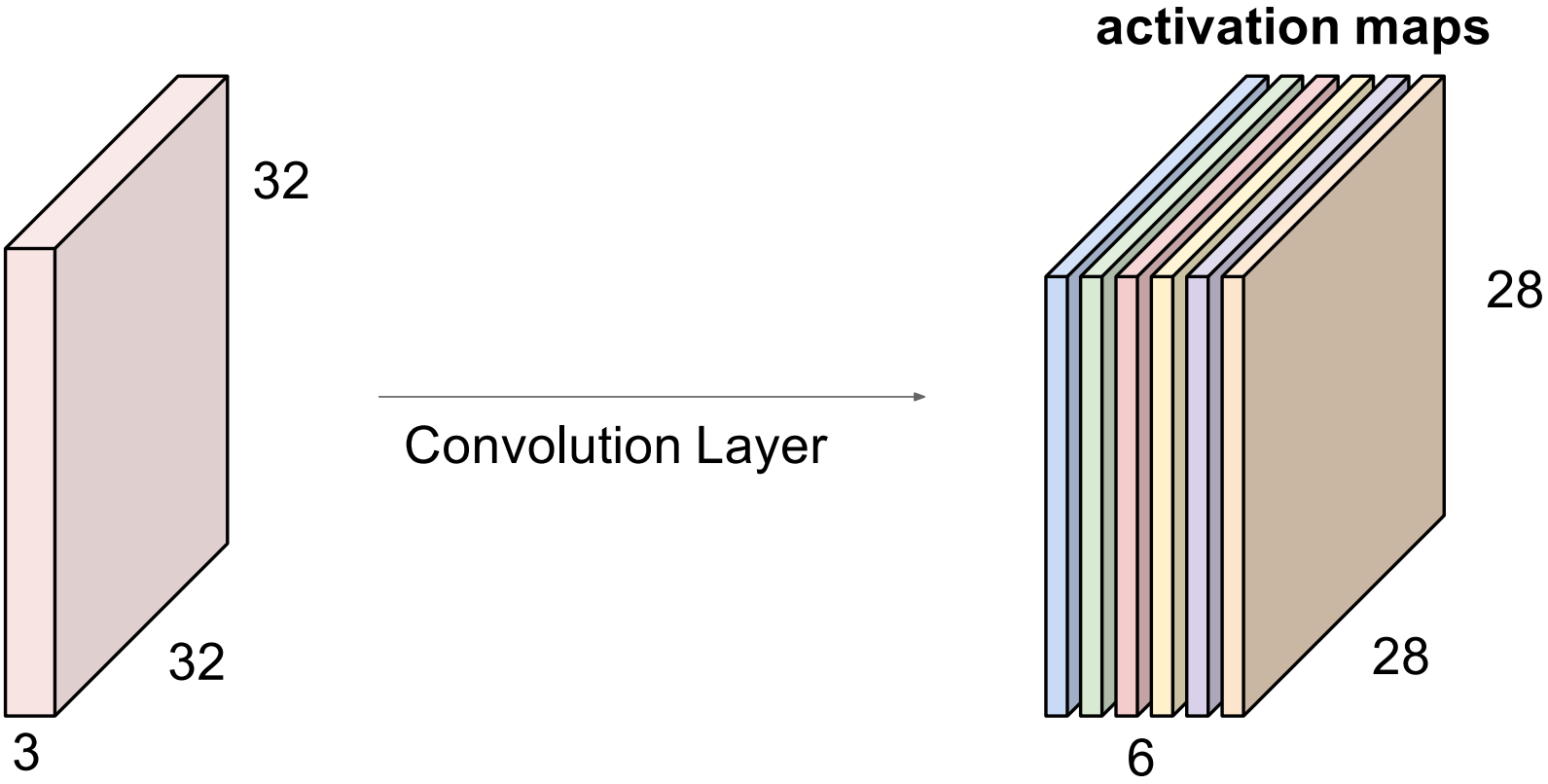


Convolution Layer

consider a second, green filter

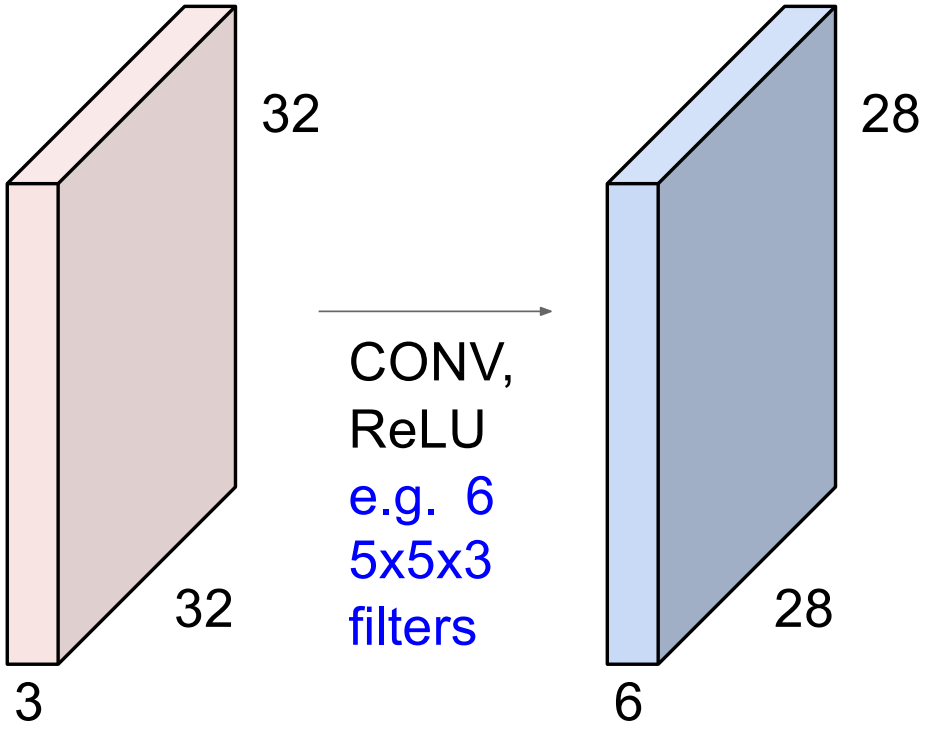


For example, if we had 6 5x5 filters, we'll get 6 separate activation maps:

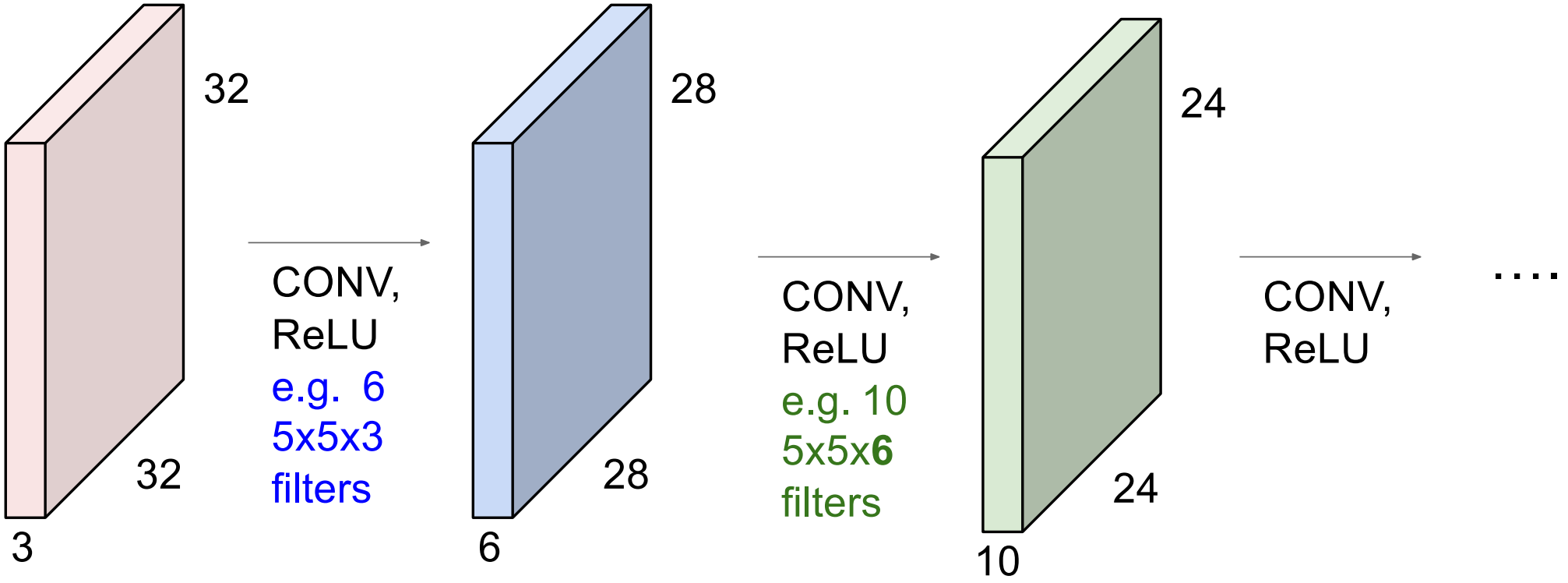


We stack these up to get a "new image" of size 28x28x6!

ConvNet is a sequence of Convolution Layers, interspersed with activation functions

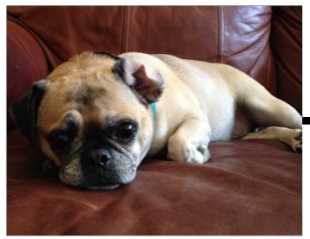


ConvNet is a sequence of Convolution Layers, interspersed with activation functions



[Zeiler and Fergus 2013]

Visualization of VGG-16 by Lane McIntosh. VGG-16 architecture from [Simonyan and Zisserman 2014].

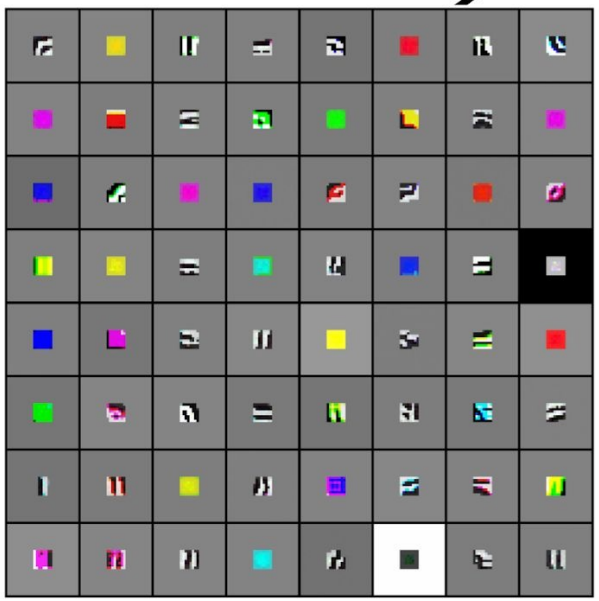


Low-level features

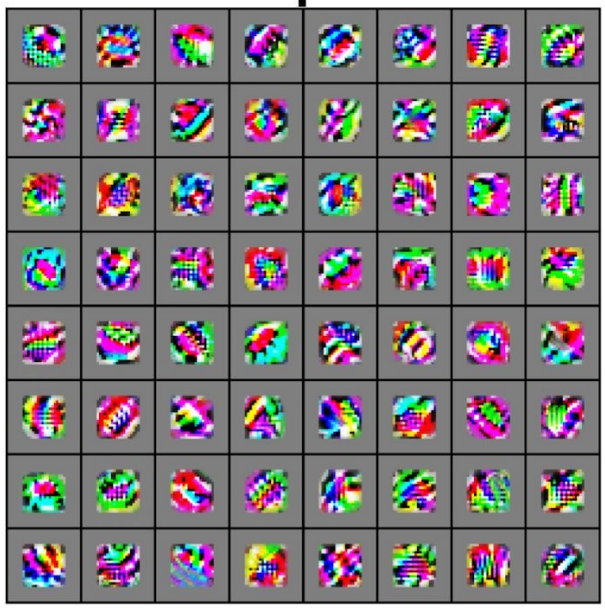
Mid-level features

High-level features

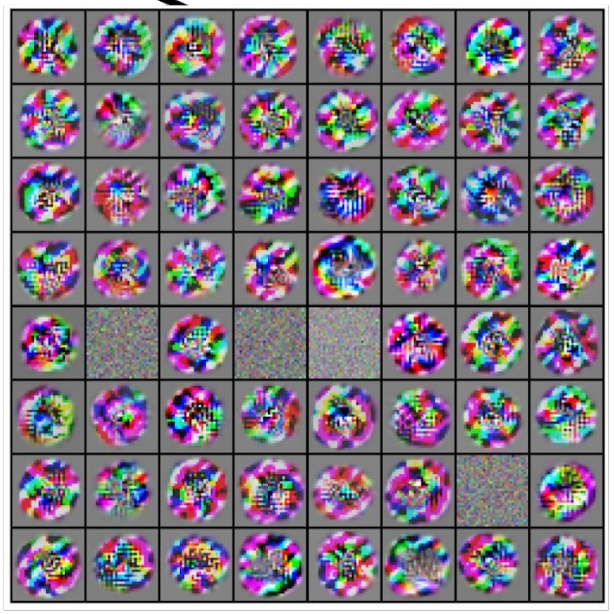
Linearly separable classifier



VGG-16 Conv1_1



VGG-16 Conv3_2



VGG-16 Conv5_3

Example: CONV layer in PyTorch

Conv layer needs 4 hyperparameters:

- Number of filters **K**
- The filter size **F**
- The stride **S**
- The zero padding **P**

Conv2d

CLASS `torch.nn.Conv2d(in_channels, out_channels, kernel_size, stride=1, padding=0, dilation=1, groups=1, bias=True)` [SOURCE]

Applies a 2D convolution over an input signal composed of several input planes.

In the simplest case, the output value of the layer with input size (N, C_{in}, H, W) and output $(N, C_{\text{out}}, H_{\text{out}}, W_{\text{out}})$ can be precisely described as:

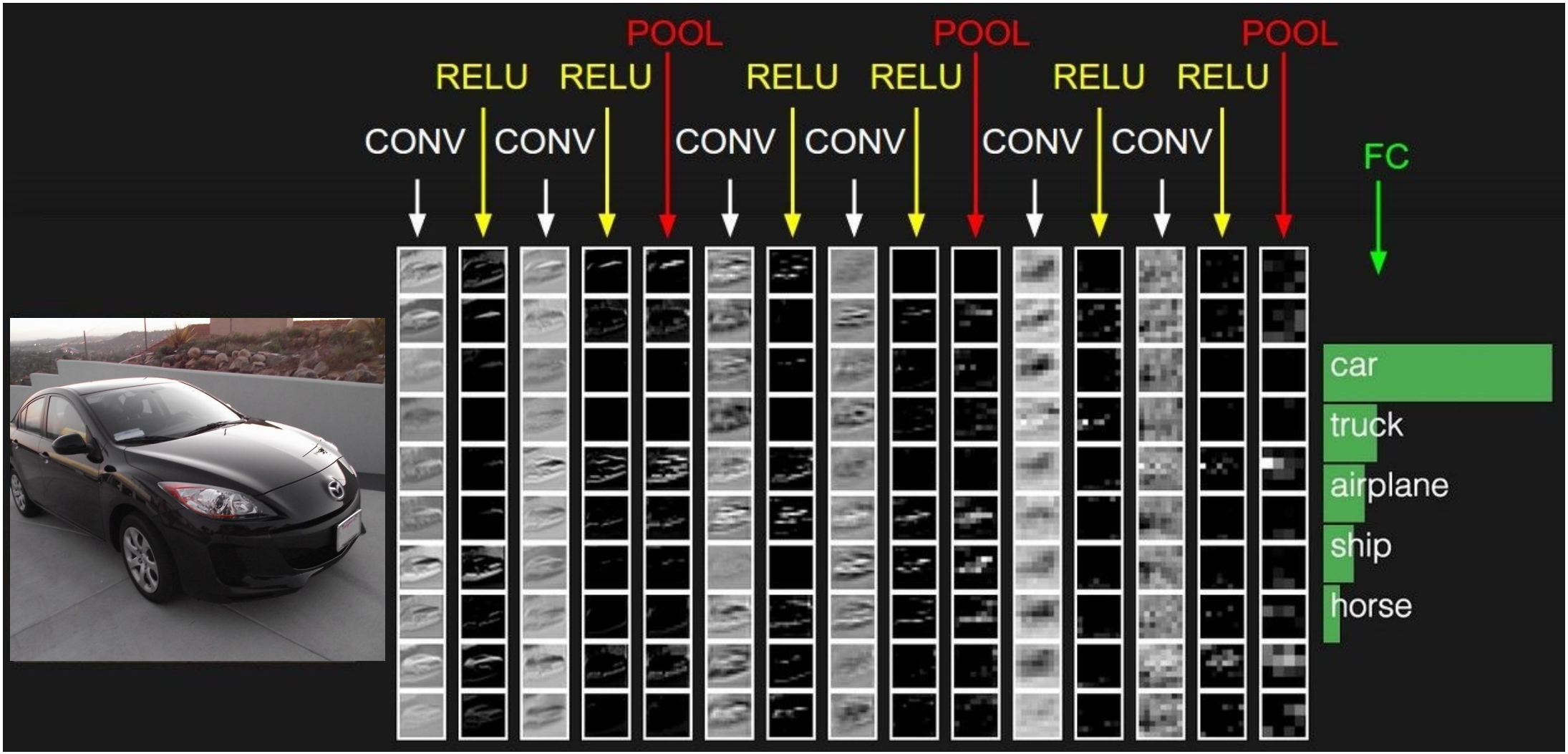
$$\text{out}(N_i, C_{\text{out}_j}) = \text{bias}(C_{\text{out}_j}) + \sum_{k=0}^{C_{\text{in}}-1} \text{weight}(C_{\text{out}_j}, k) \star \text{input}(N_i, k)$$

where $\star$ is the valid 2D cross-correlation operator, N is a batch size, C denotes a number of channels, H is a height of input planes in pixels, and W is width in pixels.

- `stride` controls the stride for the cross-correlation, a single number or a tuple.
- `padding` controls the amount of implicit zero-paddings on both sides for `padding` number of points for each dimension.
- `dilation` controls the spacing between the kernel points; also known as the à trous algorithm. It is harder to describe, but this link has a nice visualization of what `dilation` does.
- `groups` controls the connections between inputs and outputs. `in_channels` and `out_channels` must both be divisible by `groups`. For example,
  - At groups=1, all inputs are convolved to all outputs.
  - At groups=2, the operation becomes equivalent to having two conv layers side by side, each seeing half the input channels, and producing half the output channels, and both subsequently concatenated.
  - At groups= `in_channels`, each input channel is convolved with its own set of filters, of size: $\left\lfloor \frac{C_{\text{out}}}{C_{\text{in}}} \right\rfloor$.

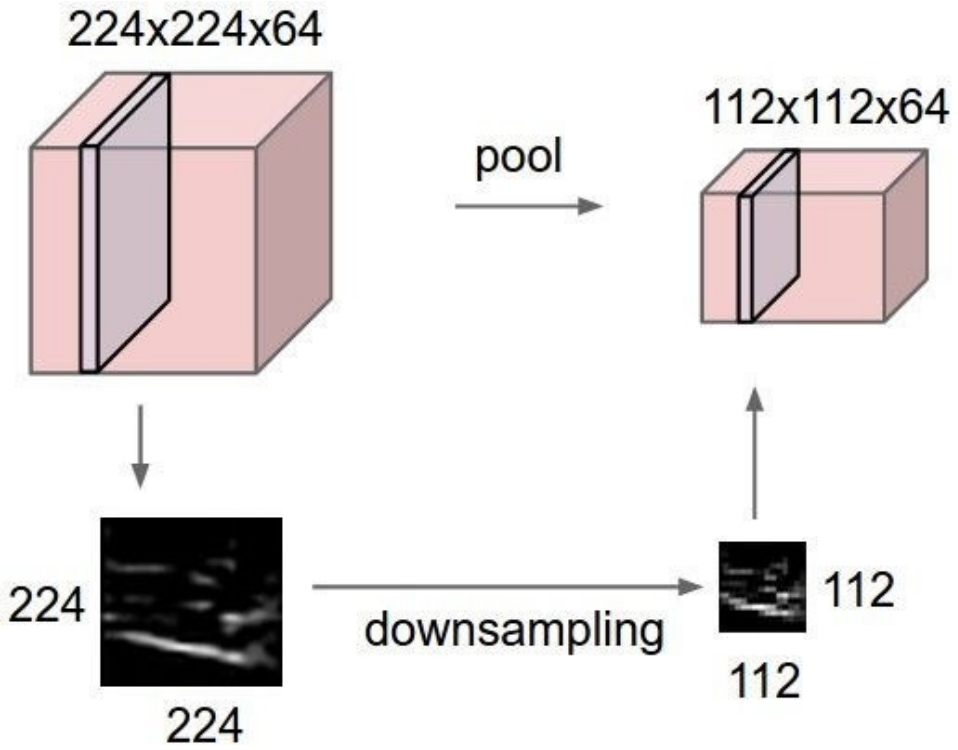
The parameters `kernel_size`, `stride`, `padding`, `dilation` can either be:

- a single `int` – in which case the same value is used for the height and width dimension
- a `tuple` of two ints – in which case, the first *int* is used for the height dimension, and the second *int* for the width dimension

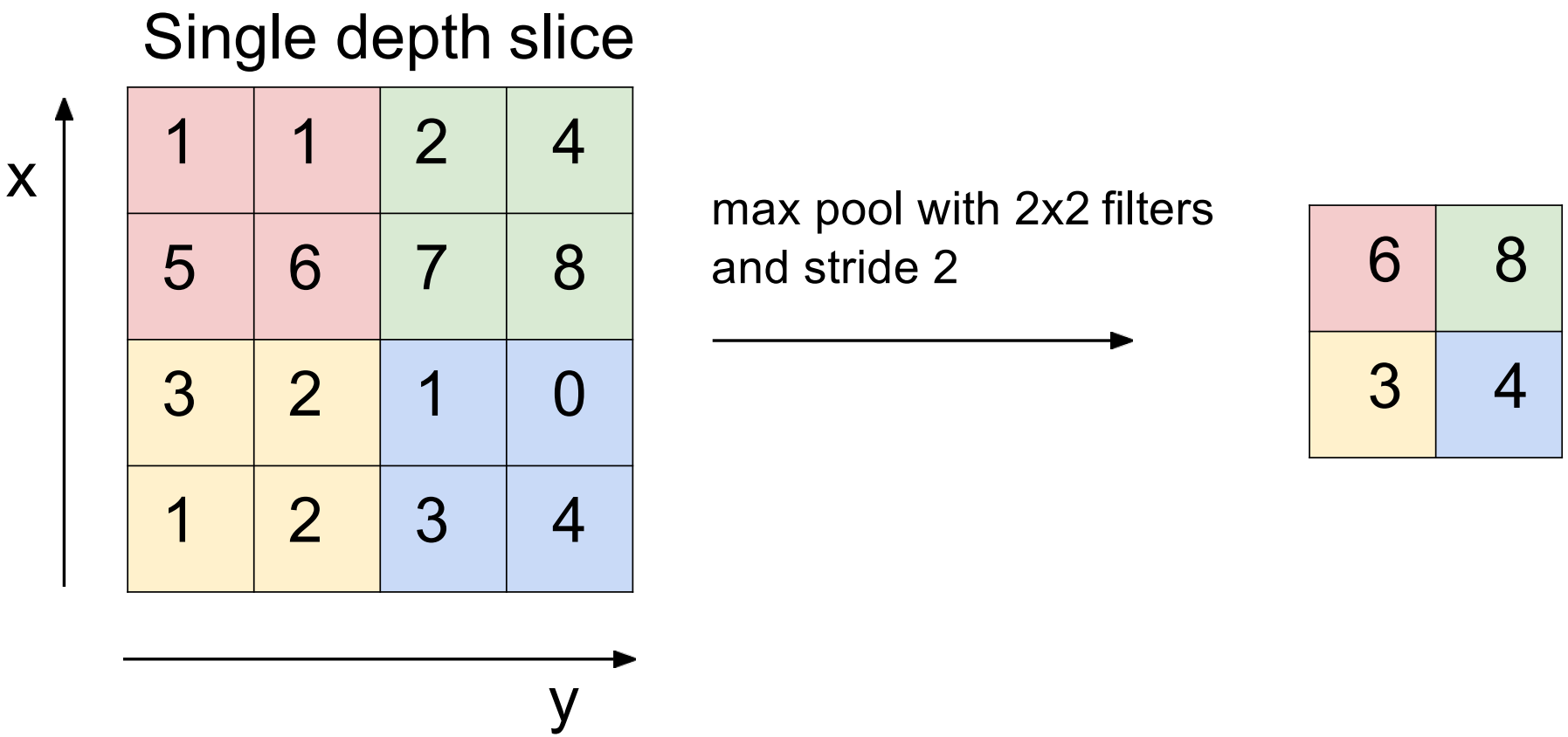

POOL
POOL
POOL
RELU
RELU
RELU
RELU
RELU
RELU
CONV
CONV
CONV
CONV
CONV
CONV
FC
car
truck
airplane
ship
horse

Pooling layer

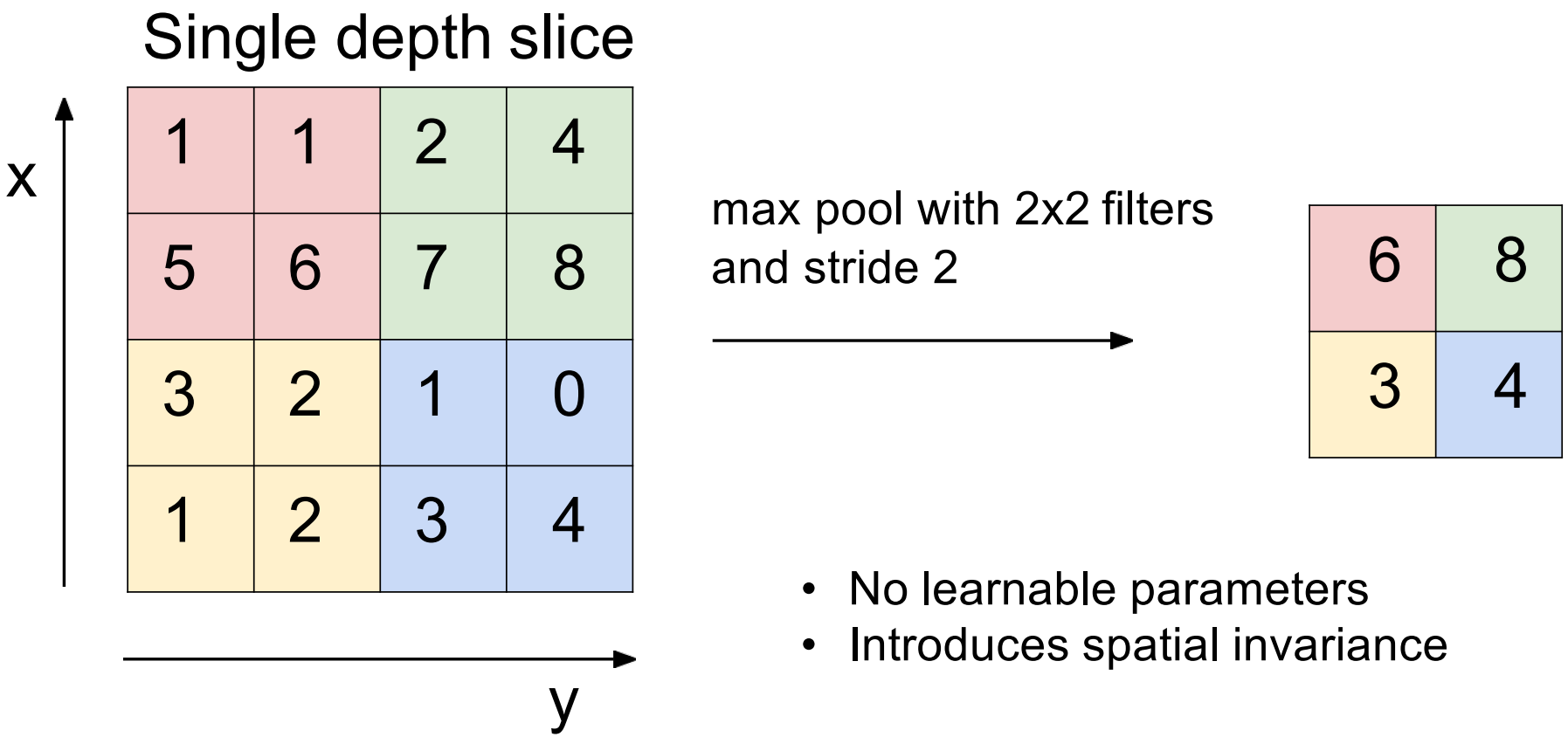
- makes the representations smaller and more manageable
- operates over each activation map independently:



MAX POOLING


Single depth slice
x
y
1 1 2 4
5 6 7 8
3 2 1 0
1 2 3 4
max pool with 2x2 filters
and stride 2
6 8
3 4

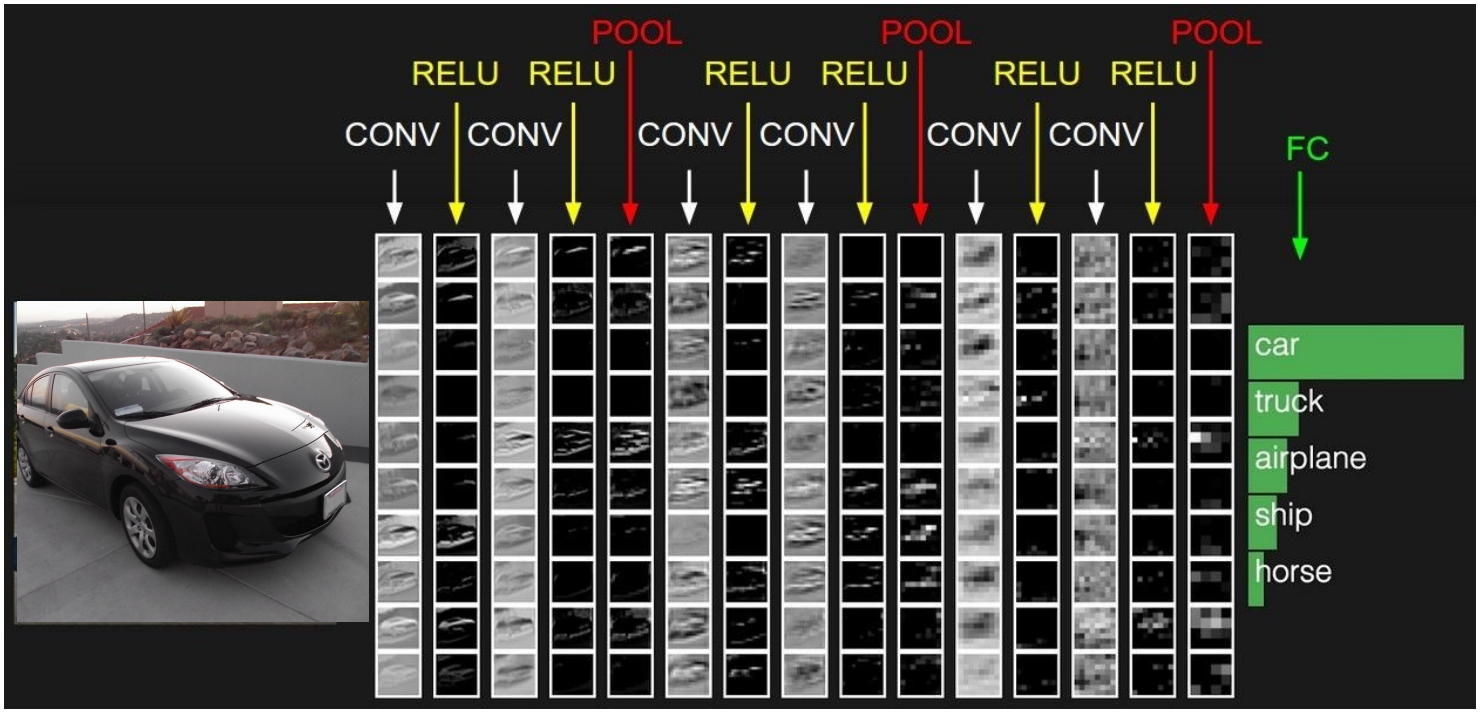
MAX POOLING



- No learnable parameters
- Introduces spatial invariance

Fully Connected Layer (FC layer)

- Contains neurons that connect to the entire input volume, as in ordinary Neural Networks



Summary

- ConvNets stack CONV,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Historically architectures looked like
  [(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K,SOFTMAX
  where N is usually up to ~5, M is large, 0 <= K <= 2.
- but recent advances such as ResNet/GoogLeNet
  have challenged this paradigm

Thank you

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